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ROBUSTNESS RESULTS FOR STATE FEEDBACK REGULATORS, (U)
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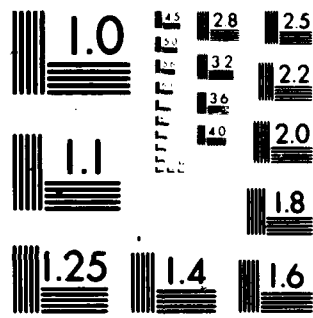
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ROBUSTNESS RESULTS FOR STATE FEEDBACK REGULATORS

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per Molander* and J. C. Willems**

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Abstract

State feedback regulators are derived which have better robustness characteristics than the standard 60° phase margin and 50% gain reduction tolerance of standard linear-quadratic regulators. It is also shown how the Lyapunov equation can be used to design high-integrity regulators for open-loop stable systems.

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1. Introduction

Any practical control system synthesis is subject to uncertainty. This uncertainty may appear as non-measurable disturbances on the input, or only partly known or time varying system parameters, to quote a few examples. The topic of the present paper is to study one example of such parameter uncertainty, namely large variations in the input channels.

The criterion chosen for our "acceptable" closed-loop system behavior is stability. This is, of course, far from sufficient for practical purposes, but such analysis may all the same produce some guidelines for the design of practical control systems. The step from a qualitative to a quantitative result is often a short one.

Since the discovery by Kalman [1] of the frequency domain inequality satisfied by linear-quadratic optimal regulators, a great deal of literature has been produced on the "robustness" of such regulators. In terms of classical control concepts, Kalman's inequality implies that they possess 60° phase margin, infinite gain margin, and 50% gain reduction tolerance. This result holds under the only assumption that the penalty matrix of the state variable in the performance index is positive semidefinite.

Although 60° phase margin seems a lot, it may be insufficient in certain cases. Dynamics of the actuators $x \rightarrow c$ and time delays may have been neglected in the model, for instance. Thus it is of interest to synthesize regulators with improved robustness characteristics.

As is easy to see from examples, the above results cannot be strengthened without further assumptions on the performance index or the plant. Assumptions used here are decreasing penalty on the control variables (cheap control) and stability of the open loop system, respectively.

2. Formulation of the Problem.

Consider the linear time invariant system described by

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t) \\ x(0) &= x_0.\end{aligned}\tag{1}$$

$x(t)$ and $u(t)$ are n - and p -vectors, respectively. Assume that the desired input is given by the linear, constant state feedback

$$u(t) = -L^T x(t)\tag{2}$$

but that the implementation is computed by some time varying nonlinearity $\phi(\cdot, t)$, i.e. the actual input $u(t)$ is given by

$$u(t) = -\phi(L^T x(t), t)\tag{2}'$$

Here, $\phi(\cdot, t)$ is a nonlinear function from R^p to R^p for each t , subject to the condition

$$K_1 \sigma^T \sigma \leq \sigma^T \phi(\sigma, t) \leq K_2 \sigma^T \sigma \quad \forall \sigma \in R^p\tag{3}.$$

The problem is to guarantee global asymptotic stability of the null solution of (1), (2)' subject to (3) for given K_1, K_2 .

Although the problem is formulated for the case of a memoryless nonlinearity $\phi(\cdot, t)$, the results can easily be generalized to dynamic disturbances using modern frequency domain stability theorems (see e.g. [2]). This has been exploited in [3], but will not be pursued here.

3. Results for Arbitrary Plants.

First a brief review of the result for linear-quadratic optimal controllers will be given. Associate with (1) the problem of optimizing, with respect to $u(\cdot)$, the performance index

$$J = \int_0^\infty (x(t)^T Q x(t) + u(t)^T R u(t)) dt.$$

It is well known that the solution is given by the linear constant feedback

$$u(t) = -R^{-1} B^T P x(t)\tag{4},$$

where P is the largest solution of the algebraic Riccati equation

For notational simplicity, $R = I_p$ will be assumed in the sequel.

Standard manipulations of equation (5) lead to the frequency domain equality

$$(I + G^T(-i\omega))I_p(I + G(i\omega)) = I_p + B^T(-i\omega - A^T)^{-1}Q(i\omega - A)^{-1}B \quad (6),$$

where

$$G(s) = R^{-1}B^TP(sI - A)^{-1}B.$$

An alternative form of (6) for the closed-loop system is, with $\tilde{G}(s) = G(s)(I + G(s))^{-1}$.

$$(I - \tilde{G}^T(-i\omega))I_p(I - \tilde{G}(i\omega)) = I_p - B^T(-i\omega - A^T)^{-1}Q(i\omega - A)^{-1}B \quad (6)'. \quad (6)'$$

If Q is positive semidefinite, the second term on the right-hand side is nonnegative, and the following inequality results:

$$(I - \tilde{G}^T(-i\omega))I_p(I - \tilde{G}(i\omega)) \leq I_p \quad (7).$$

A straight-forward application of the circle criterion then implies the desired stability for

$$K_1 = \frac{1}{2}, \quad K_2 = \text{any finite } K.$$

It is obvious that the best possible lower bound K_1 without restrictions on the plant is

$$K_1 > 0 \quad (8).$$

For certain high-gain regulators, (8) is in fact sufficient as will now be shown. Consider the performance index J_ρ , where the penalty on the input is modified via the scalar variable ρ :

$$J_\rho = \int_0^\infty (x(t)^T Qx(t) + \rho u(t)^T u(t))dt.$$

Choosing ρ small means that very large control signals are permitted. It seems reasonable to conjecture that such a design should yield a controller with high integrity. This is indeed the case.

Theorem 1. Consider the control system (1), (2), with $\phi(\cdot, t)$ subject to (3).

For any $K_1 > 0$ and any $K_2 < \infty$ there is a $Q \geq 0$ such that the optimal controller relative to J_ρ guarantees asymptotic stability in the large of the null solution for all $\rho > 0$ sufficiently small.

Proof: The Riccati equation pertaining to J_ρ is

$$A^T P_\rho + P_\rho A + Q - \rho^{-1} P_\rho B B^T P_\rho = 0 \quad (5)'$$

Two reductions of the general case will be made. Firstly, it may be assumed without loss of generality that

$$\text{rank } (Q) \leq \text{rank } (B) = p,$$

(see [4]). Thus Q can be factored as MM^T , with

$$\text{rank } (M) \leq \text{rank } (B).$$

There are several M -matrices that generate the same feedback matrix, corresponding to various factorizations of

$$B^T (-sI - A^T)^{-1} Q (sI - A)^{-1} B = B^T (-sI - A^T)^{-1} MM^T (sI - A)^{-1} B.$$

If the M that makes

$$M(s) = M^T (sI - A)^{-1} B$$

minimum phase is chosen, the conditions are satisfied which ensure that ([5], [6])

$$\lim_{\rho \rightarrow 0} \frac{P_\rho}{\rho} = 0$$

Equivalently,

$$\lim_{\rho \rightarrow 0} \rho^{-1} P_\rho B B^T P_\rho = Q = MM^T \quad (9).$$

Starting from Riccati equation (5)' and using the same manipulations that lead to inequality (6)' then shows, inserting (9), that

$$\tilde{G}(s) = \rho^{-1} B^T P_\rho (sI - A + \rho^{-1} B B^T P_\rho)^{-1} B$$

satisfies

$$\left(\frac{1}{2}I - \tilde{G}_\rho^T(-s)\right) I_p \left(\frac{1}{2}I - \tilde{G}_\rho(s)\right) \leq I_p + O(\rho^{-1}), \quad \rho \rightarrow 0.$$

This proves the claim.

Remark: A weaker version of Theorem 1, corresponding to a $\phi(\cdot, t)$ that is

diagonal (i.e. no cross-couplings between the inputs), can be proved using the results of [7].

In classical terminology, this regulator possesses a phase margin arbitrary close to 90° , infinite gain margin, and a gain reduction tolerance arbitrarily close to 100%.

There is an alternative way of generating such controllers, which is almost trivial. The proof is left to the reader.

Theorem 2. Consider (1), (2)' with $\phi(\cdot, t)$ subject to (3). If L is chosen as

$$L = \epsilon^{-1} L_0,$$

where L_0 is any optimal gain generated from a positive semidefinite Q , then the controller enjoys the same robustness properties when $\epsilon \rightarrow 0$ as the controller of Theorem 1 when $\rho \rightarrow 0$.

Theorems 1 and 2 must be used with some caution, since in applications the sector bounds K_1 and K_2 may depend on the nominal feedback gain L . An example is given in Section 5.

4. Results for Open-Loop Stable Plant

If the plant is open-loop stable, it should be possible to design controllers where the lower bound K_1 is zero. A complete characterization of such feedback matrices L is given in the following theorem.

Theorem 3. The feedback system (1), (2)' with $\phi(\cdot, t)$ satisfying (3) has a globally asymptotically stable equilibrium solution for

$$K_1 = 0, \quad K_2 = \text{any } K > 0$$

if there exist matrices $K = K^T > 0$ and C , (C, A) being an observable pair, such that

$$\begin{cases} A^T K + KA = -CC^T \\ KB = L. \end{cases} \quad (10)$$

Proof: This is a straight-forward application of the Kalman Yakubovich-Popov lemma in its multivariable form ([8]). Equation (10) provides a means to design high-integrity controllers, namely by solving the (linear) Lyapunov equation for K and then choosing the feedback gain L as (any positive multiple of) KB. There remains the problem of choosing C and a positive gain, but this problem is shared by the LQOC methodology. For single-input systems, equation (10) is simplified further if it is translated into the frequency domain. Defining $p(s)$ as the open-loop characteristic polynomial and

$$\frac{q(s)}{p(s)} = L^T (sI - A)^{-1} B$$

$$\frac{r(s)}{p(s)} \frac{r(-s)}{p(-s)} = B^T (-sI - A^T)^{-1} C C^T (sI - A)^{-1} B$$

yields the equation

$$p(s) q(-s) + q(s) p(-s) = r(s) r(-s) \quad (10)'$$

This can be solved for $q(s)$, which in turn gives the unique feedback gain L.

The condition of Theorem 3 is rather strong. In general an infinite gain margin is not required. The following theorem characterizes the feedback gains that retain stability for all gain drops.

Theorem 4. The claim of Theorem 3 remains valid for

$$K_1 = 0, \quad K_2 = 1$$

if there exist matrices $K = K^T > 0$, C, and D, and an $\epsilon > 0$ such that

$$\begin{aligned} A^T K + K A &= -C C^T - D D^T \\ K B &= 1 - (\sqrt{2} - \epsilon) D \end{aligned} \quad (11)$$

Proof. This is again a direct application of the circle criterion in combination with the Kalman-Yakubovich-Popov lemma. Compared to equation (10), the use of equation (11) involves the extra problem of choosing D. Theorem 4 can of course

be generalized to any upper and lower bounds K_1 and K_2 (provided the system is open loop asymptotically stable if $K_1 \leq 0 \leq K_2$).

5. Discussion of the Results

The results presented above must be interpreted with some caution. It is quite clear that most actuator failures are such that no judicious choice of feedback gains can save the situation. This is even more true for the case of sensor failures (not treated here). Thus in general one will have to rely on external failure-handling routines.

The sharpening of previous results for arbitrary plants relies upon the use of high gains. This must be kept in mind if the nominal input influences the sector of $\phi(\cdot, t)$. For instance, if the gain drop of the regulator is caused by saturation, there is obviously no point in increasing the nominal gain.

With these reservations kept in mind, the theorems yet contribute to the general robustness picture of state feedback regulators. Theorem 3 is believed to yield a practicable design method for control systems where the input channel uncertainty is so great that it has to be accounted for in the synthesis.

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